

Name: ANSWERS

Instructor: Bullwinkle

Math 10120, Exam I.
October 14, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators **are** allowed
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 12 pages of the test.

| PLEASE MARK YOUR ANSWERS WITH AN X, not a circle! | | | | |
|---------------------------------------------------|-----|-----|-----|---------|
| 1. | (a) | (b) | (●) | (d) (e) |
| 2. | (a) | (b) | (c) | (●) (e) |
| | | | | |
| 3. | (a) | (b) | (c) | (d) (●) |
| 4. | (●) | (b) | (c) | (d) (e) |
| | | | | |
| 5. | (a) | (b) | (c) | (●) (e) |
| 6. | (a) | (b) | (c) | (●) (e) |
| | | | | |
| 7. | (a) | (b) | (c) | (d) (●) |
| 8. | (a) | (●) | (c) | (d) (e) |
| | | | | |
| 9. | (a) | (b) | (c) | (d) (●) |
| 10. | (●) | (b) | (c) | (d) (e) |

| |
|-----------------------------------------|
| Please do NOT write in this box. |
| Multiple Choice _____ |
| 11. _____ |
| 12. _____ |
| 13. _____ |
| 15. _____ |
| Total _____ |

3.

Initials: _____

3.(5pts) A sample space consists of 7 simple outcomes $\{a, b, c, d, e, f, g\}$. The probabilities are

| $P(a)$ | $P(b)$ | $P(c)$ | $P(d)$ | $P(e)$ | $P(f)$ | $P(g)$ |
|--------|--------|--------|--------|--------|--------|--------|
| 0.10 | 0.06 | 0.15 | 0.09 | 0.08 | 0.12 | 0.40 |

What is $P(\{e, c, f\})$?

- (a) 0.65 (b) $\frac{3}{7}$ (c) 0.00144 (d) 0.429 (e) 0.35

4.(5pts) Suppose $P(E) = 0.4$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$. Which of the statements below is correct?

- (a) E and F are neither independent nor mutually exclusive.
(b) E and F are independent but not mutually exclusive.
(c) Neither Independence nor exclusivity can be determined from the given data.
(d) E and F are mutually exclusive but not independent.
(e) E and F are independent and mutually exclusive.

4.

Initials: _____

5.(5pts) Suppose that E and F are events in an experiment, and $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$, $P(E \cup F) = \frac{3}{4}$. What is $\Pr(E|F)$.

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) 0

(e) 1

6.(5pts) A new piece of electronic equipment has five components. the probability of failure within a year is 0.1 for each component. Assuming that the failure of the various components are independent of each other, what is the probability that no component will fail in the first year?

(a) $(0.1)^5$

(b) 0.5

(c) $1 - (0.1)^5$

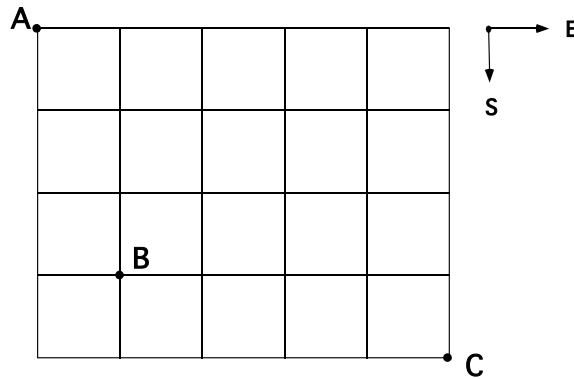
(d) $(0.9)^5$

(e) $1 - (0.9)^5$

5.

Initials: _____

- 7.(5pts) A street map of Mathland is shown below. If a taxi driver chooses a random route from A to C traveling south and east only, what is the probability that he will **not pass through the intersection at B** ?



(a) $1 - \frac{4}{126}$

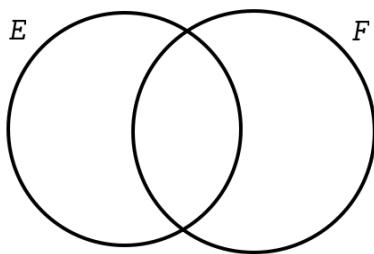
(b) $\frac{4}{126}$

(c) $1 - \frac{5}{126}$

(d) $\frac{20}{126}$

(e) $1 - \frac{20}{126}$

- 8.(5pts) Suppose given two events, E and F , such that $P(E) = 40\%$, $P(F) = 60\%$ and $P(F|E) = 33\frac{1}{3}\%$. What is $P(E \cup F)'$?



(a) 10%

(b) 20%

(c) 40%

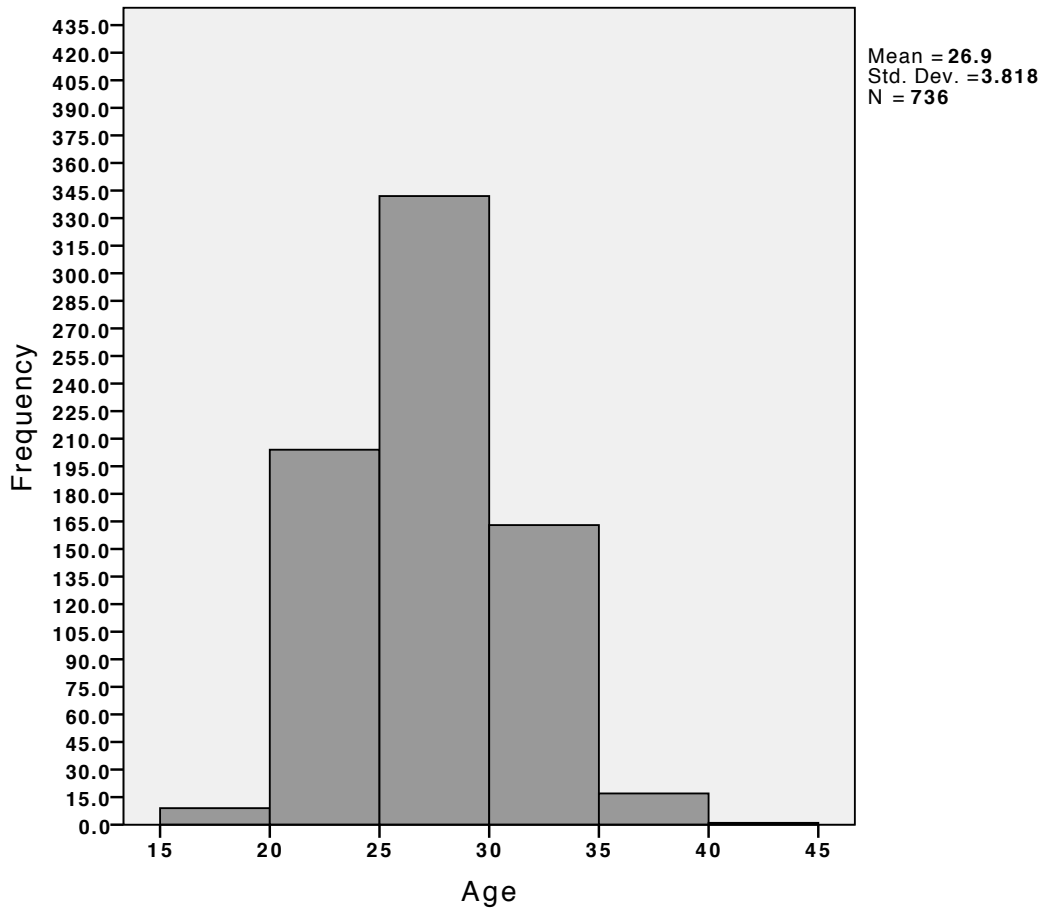
(d) 50%

(e) 30%

6.

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9.(5pts) The histogram shown below, gives the frequency of age groups for all players in the World Cup of 2014. There were 736 players in the 2014 World Cup. (We assume that the histogram follows the convention that data at a boundary of categories goes in the category on the right.)



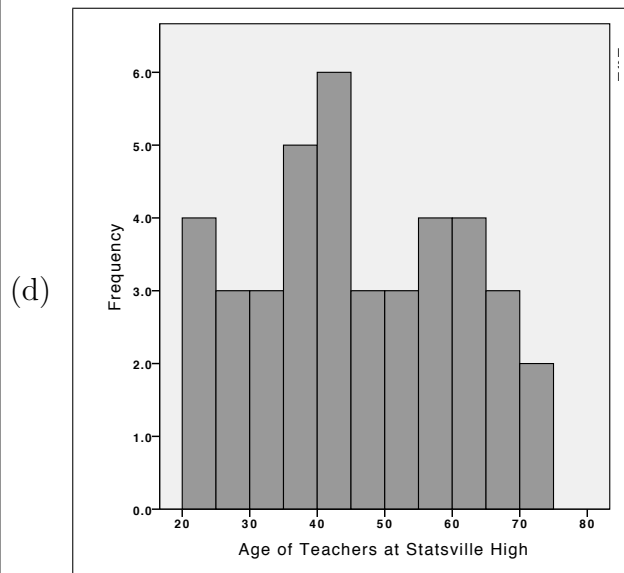
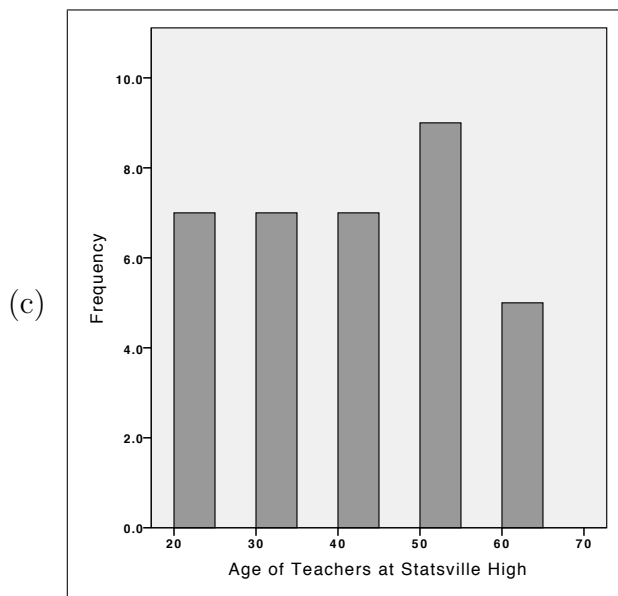
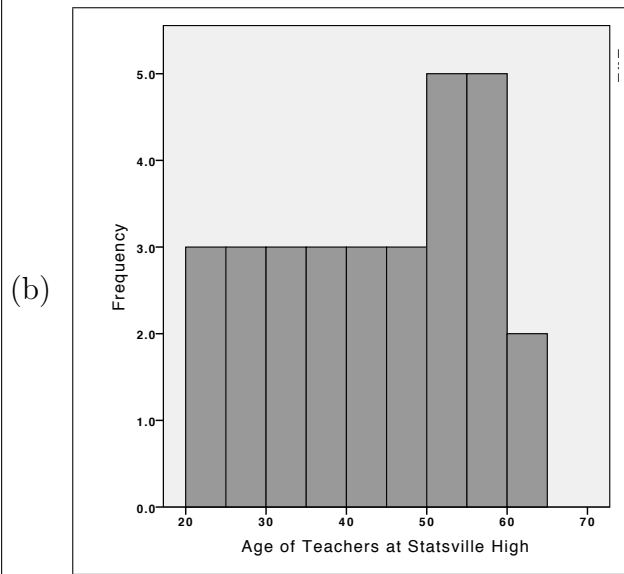
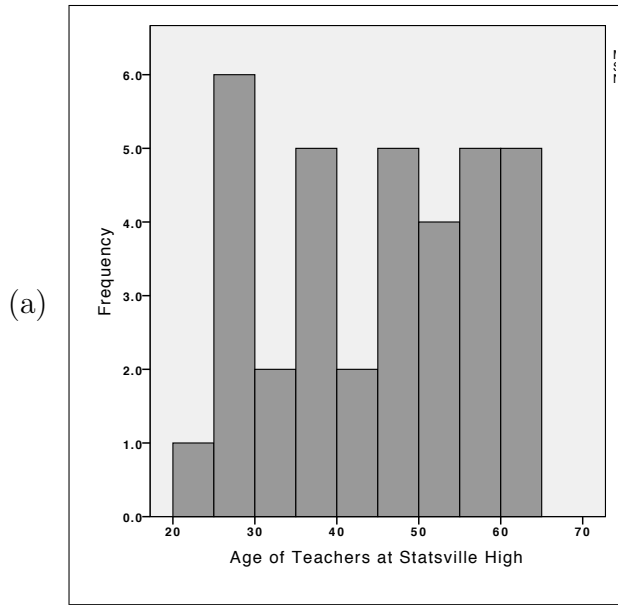
Which of the following statements can be deduced from the information given in the histogram?

- (a) More than one three quarters of the players were 25 years old or older.
- (b) More than 150 players were exactly 32.5 years old
- (c) At least one player was 15 years old.
- (d) One player was 45 years old.
- (e) At least 40% of the players were in the age bracket 25-30 (age in the interval $[25, 30)$).

10.(5pts) The data given in the following stem and leaf plot shows the ages of all teachers at Statsville High School.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 5 | 5 | 9 | 9 | 9 | 9 | |
| 3 | 0 | 0 | 5 | 5 | 7 | 7 | 9 | |
| 4 | 0 | 0 | 5 | 5 | 5 | 9 | 9 | |
| 5 | 0 | 0 | 1 | 4 | 6 | 7 | 8 | 8 |
| 6 | 0 | 1 | 2 | 3 | 4 | | | |

Which of the following gives a histogram of the data in the stem and leaf plot above.



(e) None of the above

8.

Initials: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

Where applicable, answers may be given in the form of products of numbers and symbols for factorials and numbers of permutations and combinations.

11.(12pts) Flip a fair coin 15 times and record the sequence of heads and tails. *Write your answers to the following questions using combinations of powers, permutations, combinations or factorials as appropriate.*

(a) What is the probability that we get exactly one head?

(b) What is the probability that we get exactly two heads?

(c) What is the probability that we get exactly five heads?

(d) What is the probability that we get at least 3 heads?

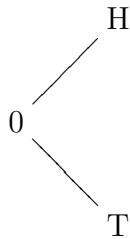
9.

Initials: _____

12.(12pts) Imagine a 6 sided die but instead of the numbers from 1 to 6 on the faces, only the numbers 1 and 2 appear on the faces: 1 occurs twice and 2 occurs 4 times. Hence if you roll one of these die and record the number on top the probability that you get a 1 is $\frac{1}{3}$ and the probability that you get a 2 is $\frac{2}{3}$.

Consider the following game. First you flip a fair coin. If you get a head, you roll one of the die discussed in the last paragraph and then you roll it again. If you get a tail you roll only once.

(a) Draw a tree diagram for this game and fill in the probabilities. Let H denote the event that you flipped a head and let T denote the event that you flipped a tail. Let 1 denote the event that you rolled a 1 and 2 denote the event that you rolled a 2. The first step in the diagram is given below.



(b) Using your tree diagram calculate the probability that you get exactly one 2 when you play this game once.

10.

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- 13.**(12pts) John, Paul and Luke are responsible for the output of decanters at a glass factory. The table below shows the proportion of the output for which each is responsible and the probability that a decanter chosen at random from their respective outputs is defective. If a decanter shipped to Notre Dame bookstore is defective, what is the probability it was produced by John?

| Worker | Proportion of Output | Pr(defective) |
|--------|----------------------|---------------|
| John | 0.5 | 0.1 |
| Paul | 0.3 | 0.5 |
| Luke | 0.2 | 0.7 |

11.

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14.(12pts) A random sample of 1000 people was chosen from the Population of Iseland. For each person in the sample, the eye color and age was recorded. The results are shown in the table below.

| | | Eye Color | | | Totals |
|--------|------------|-----------|-------|-------|--------|
| | | Blue | Green | Brown | |
| Age | 65 or over | 100 | 90 | 70 | 260 |
| | 40 - 64 | 60 | 120 | 60 | 240 |
| | 25 - 39 | 90 | 110 | 50 | 250 |
| | 0 - 24 | 115 | 110 | 25 | 250 |
| Totals | | 365 | 430 | 205 | 1000 |

The record for one of the people from the sample is chosen at random.

Let B be the event that the record chosen is that of someone with blue eyes, let S be the event that the record chosen is that of someone aged 65 or older.

(a) What is the probability that the record chosen is that of someone with Blue eyes, $P(B)$?

(b) What is the probability that the record chosen is that of someone who has blue eyes and is aged 65 or older, $P(B \cap S)$?

(c) What is $P(B \cup S)$, that is the probability that the record chosen is that of someone who has blue eyes or is aged 65 or over or both.

(d) What is $P(B|S)$, that is the probability that the record chosen is that of someone with blue eyes, given that it is a record of someone aged 65 or older?

(e) Are B and S independent events? Justify your answer.

12.

Initials: _____

15.(2pts) You will get this 2 points if your instructor can read your name easily on the front page of the exam and you mark the answer boxes with an X (as opposed to a circle or any other mark).